Engineering Notes

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Generic Model of a Large Flexible Space Structure for Control Concept Evaluation

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Introduction

IT is generally recognized that one of the technological challenges posed by a large space structure is the control of its orientation (attitude) and, in some applications, its shape. These control problems are exacerbated by the significant flexibility of the structure. Translation and attitude motions are coupled with structural vibrations. To achieve intended control system performance, sensors and actuators must be placed at appropriate locations on the structure. This requirement is most evident for shape control and suggests the application of multivariable control theory. Unfortunately, it is difficult to compare the various control methods proposed for multivariable control unless each is illustrated by the same nontrivial generic problem. The main purpose of this paper is to present what we believe is a model sufficiently general yet sufficiently simple to serve as a standard on which to test new control concepts and algorithms. In other words, our aim is to describe dynamic characteristics and control specifications for the shape and attitude control of a representative large space structure. Reduction of the finite-element model is based on *modal identities* and *modal cost analysis*.

Generic Model

As stated above, the model offered here is on the shape control of $\mathcal{E}$ and the attitude control of $\mathcal{R}$ (see Fig. 1). This is expressed in the following performance index:

$$V=\lim_{t\to\infty} \mathrm{E}\left[\int_{\mathcal{V}} \{w^2(x,y,t)+\beta\dot{w}^2(x,y,t)\}\mathrm{d}m+\boldsymbol{u}^T(t)\boldsymbol{R}\boldsymbol{u}(t)\right] \tag{1}$$

where E is an expectation operator, $\mathcal{V}=\mathcal{R}+\mathcal{E}$, $w(x,y,t)$ is the displacement with respect to the inertial frame (x,y,z) at any point P of the vehicle, β is a weighting scalar on the displacement rate $\dot{w}$, $\mathrm{d}m$ is element mass, and $\boldsymbol{R}$ is a weighting matrix on the control vector $\boldsymbol{u}(t)$. Superscript T denotes transpose. To separate the attitude control of $\mathcal{R}$ from the shape control of $\mathcal{E}$, we first note that

$$w(x,y,t)=z_c(t)+y\theta_x(t)-x\theta_y(t)+\begin{cases}0 & P\in\mathcal{R}\\ u(x,y,t) & P\in\mathcal{E}\end{cases} \tag{2}$$

where (z_c,θ_x,θ_y) are the rectilinear motion along the z axis and rotation about the x and y axes, respectively, of the rigid body $\mathcal{R}$; $u(x,y,t)$ is the deformation of $\mathcal{E}$. Using Eq. (2), we deduce

$$\int_{\mathcal{V}} w^2(x,y,t)\mathrm{d}m=(m_r z_c^2+I_{rx}\theta_x^2+I_{ry}\theta_y^2)+\int_{\mathcal{E}} w^2(x,y,t)\mathrm{d}m \tag{3}$$

where m_r, I_{rx}, I_{ry} define mass and moments of inertia of $\mathcal{R}$ about the x and y axes, respectively. The rigid-body motion and the elastic deformation can be expressed in terms of the *vehicle* modal coordinates η as follows.

$$\begin{bmatrix} z_c(t)\\ \theta_x(t)\\ \theta_y(t)\\ u(x,y,t)\end{bmatrix}=\begin{bmatrix} z_{c1}\eta_1(t)\\ \theta_{x2}\eta_2(t)\\ \theta_{y3}\eta_3(t)\\ 0\end{bmatrix}+\Sigma'_\alpha\begin{bmatrix} z_{c\alpha}\\ \theta_{x\alpha}\\ \theta_{y\alpha}\\ u_\alpha(x,y)\end{bmatrix}\eta_\alpha(t)$$

$$\Sigma'_\alpha=\sum_{\alpha=4}^{\infty} \tag{4}$$

where $\eta_\alpha\,(\alpha=1,2,3)$ are the 'rigid' modal coordinates and $\eta_\alpha(\alpha=4,\ldots)$ are 'elastic' modal coordinates; $z_{c\alpha}$, $\theta_{x\alpha},\theta_{y\alpha}\,(\alpha=4,\ldots)$ are the measures of participation of the rigid body $\mathcal{R}$ in the vehicle mode w_α defined as

$$w_\alpha(x,y)=z_{c\alpha}+y\theta_{x\alpha}-x\theta_{y\alpha}+u_\alpha(x,y) \qquad (\alpha=4,\ldots) \tag{5}$$

$u_\alpha\,(\alpha=4,\ldots)$ is the deformation of the structure $\mathcal{E}$ in mode α.

To avoid ill-conditioning of computations we will adopt nondimensionalization. In the following, the tilde represents a nondimensional quantity and the open circle is derivative with respect to the nondimensional time τ. Employing Eq. (3) and its rate companion, Eq. (1) can be nondimensionally rewritten such that the rigid-body motion is separate from the structural motion; thus,

$$\tilde{V}=\lim_{\tau\to\infty} \mathrm{E}[\tilde{m}_r(\tilde{z}_c^2+\tilde{\beta}\mathring{\tilde{z}}_c^2)+\tilde{I}_{rx}(\theta_x^2+\tilde{\beta}\mathring{\theta}_x^2)+\tilde{I}_{ry}(\theta_y^2+\tilde{\beta}\mathring{\theta}_y^2)$$

$$+\int_{\mathcal{E}}\{\tilde{w}^2+\tilde{\beta}\mathring{\tilde{w}}^2\}\frac{\mathrm{d}x}{a'}\frac{\mathrm{d}y}{b'}+\tilde{u}^T(\tau)\tilde{R}\tilde{u}(\tau)] \qquad \begin{matrix}a'=a/4\\ b'=b/4\end{matrix} \tag{6}$$

To conserve space, relations between dimensional and nondimensional quantities will not be displayed here; however, to derive maximum benefits from the model the reader is urged to consult Ref. 1. The angles θ_x, θ_y are in radians and are not nondimensionalized. Using the nondimensional version of Eq. (4), the rigid-body motion can be related to the modal coordinates as follows:

$$\lim_{\tau\to\infty} \mathrm{E}(\tilde{z}_c^2,\theta_x^2,\theta_y^2)$$

$$=\mathrm{tr}[\{\tilde{Z}_c\tilde{Z}_c^T,(\sigma^2N_e^2)\tilde{\Theta}_x\tilde{\Theta}_x^T,N_e^2\tilde{\Theta}_y\tilde{\Theta}_y^T\}\mathrm{E}(\tilde{\eta}\tilde{\eta}^T)] \tag{7}$$

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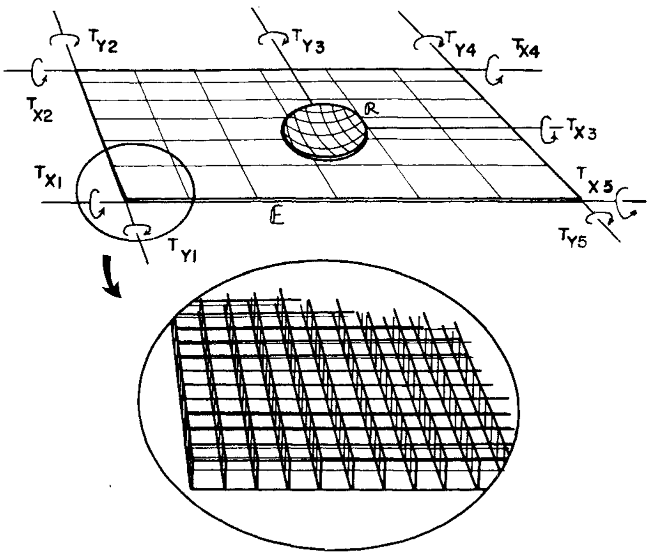

Fig. 1 A large two-dimensional space vehicle.

where

$$\tilde{Z}_c^T = [\tilde{z}_{c1}, 0, 0 \vdots \tilde{z}_e^T],\ \tilde{\Theta}_x^T = [0, \tilde{\theta}_{x2}, 0 \vdots \tilde{\theta}_{xe}^T],\ \tilde{\Theta}_y^T = [0, 0, \tilde{\theta}_{y3} \vdots \tilde{\theta}_{ye}^T] \tag{8}$$

$$\tilde{z}_e^T = [\tilde{z}_{c4}, \tilde{z}_{c5}, \dots],\ \tilde{\theta}_{xe}^T = [\tilde{\theta}_{x4}, \tilde{\theta}_{x5}, \dots],\ \tilde{\theta}_{ye}^T = [\tilde{\theta}_{y4}, \tilde{\theta}_{y5}, \dots]$$

$$N_e = 4 \qquad \sigma = 2.5$$

For the structural deformations in Eq. (6) it can be shown that

$$\lim_{\tau\to\infty} \mathrm{E}\int_{\mathcal{E}} \tilde{w}^2 \frac{\mathrm{d}x}{a'} \frac{\mathrm{d}y}{b'} = \mathrm{tr}([*]\cdot \mathrm{E}[\tilde{\eta}\tilde{\eta}^T]) \tag{9}$$

where [*] =

$$\left[\begin{array}{ccc|c} \dfrac{m_e}{m} & 0 & 0 & -N_e^2 \dfrac{m_r}{m_e} \tilde{z}_{c1} \tilde{z}_e^T \\ 0 & \dfrac{I_{ex}}{I_{xx}} & 0 & \dfrac{-N_e^4 I_{rx}}{12 I_{ex}} \tilde{\theta}_{x2} \tilde{\theta}_{xe}^T \\ 0 & 0 & \dfrac{I_{ey}}{I_{yy}} & \dfrac{-N_e^4 I_{ry}}{12 I_{ey}} \tilde{\theta}_{y3} \tilde{\theta}_{ye}^T \\ \hline -N_e^2 \dfrac{m_r}{m_e} \tilde{z}_{c1} \tilde{z}_e, & -\dfrac{N_e^4 I_{rx}}{12 I_{ex}} \tilde{\theta}_{x2} \tilde{\theta}_{xe}, & -\dfrac{N_e^4 I_{ry}}{12 I_{ey}} \tilde{\theta}_{y3} \tilde{\theta}_{ye} & 1 - [**] \end{array}\right] \tag{10}$$

$$[**] = N_e^2 \frac{m_r}{m_e} \tilde{z}_e \tilde{z}_e^T + \frac{N_e^4}{12} \left(\frac{I_{rx}}{I_{ex}} \tilde{\theta}_{xe} \tilde{\theta}_{xe}^T + \frac{I_{ry}}{I_{ey}} \tilde{\theta}_{ye} \tilde{\theta}_{ye}^T \right) \tag{11}$$

$$m_r/m_e = 0.1,\ I_{rx}/I_{ex} = I_{ry}/I_{ey} = 0.01,\ \alpha = 12.5\ \mathrm{km},$$
$$b = 5.0\ \mathrm{km},\ m = m_r + m_e,\ I_{xx} = I_{rx} + I_{ex},\ I_{yy} = I_{ry} + I_{ey} \tag{12}$$

Note that (m_e, I_{ex}, I_{ey}) and (m, I_{xx}, I_{yy}) represent mass and moments of inertia about the x and y axes of $\mathcal{E}$ and $\mathcal{V}$, respectively. In Eqs. (7-12) and in the sequel the modal-cost-mode-sequence of Skelton et al.[2] is intended. **1** is an identity matrix of appropriate size. The controlled behavior of η, $\mathring{\eta}$ predicted by any control algorithm under scrutiny can be substituted in Eqs. (7) and (9) and in their rate companions, and the attitude control of $\mathcal{R}$ and the flatness control of $\mathcal{E}$ be studied separately.

The nondimensional generic model in state space form is as follows:

$$\mathring{x} = Ax + Bu + Dw, \quad w = \mathfrak{N}(0, Q_w)$$

$$y = Px$$

$$z = Mx + v, \quad v = \mathfrak{N}(0, Q_v) \tag{13}$$

where

$$x = \begin{bmatrix} \tilde{\eta}_r \\ \mathring{\tilde{\eta}}_r \end{bmatrix},\ A = \begin{bmatrix} 0 & 1_r \\ -\tilde{\omega}_r^2 & -2\tilde{\zeta}_r\tilde{\omega}_r \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \tilde{\mathcal{B}}_r \end{bmatrix},\ D = \begin{bmatrix} 0 \\ \tilde{\mathfrak{D}}_r \end{bmatrix}$$

$$P = \begin{bmatrix} 1_r & 0 \\ 0 & 1_r \end{bmatrix},\quad M = [\tilde{\mathfrak{M}}_r \quad 0],\quad \tilde{y}^T = [\tilde{\eta}_r^T \quad \mathring{\tilde{\eta}}_r^T]$$

$$\tilde{u}^T = [T_{x1} T_{y1} T_{x2} T_{y2} T_{x3} T_{y3} T_{x4} T_{y4} T_{x5} T_{y5}] \tag{14}$$

Let the number of modes retained be r, including the two rigid modes η_2, η_3 from Eq. (4); presently, $r = 11$. η_r denotes an r-vector of vehicle-modal coordinates; ω_r and ζ_r are $r\times r$ matrices of the natural frequencies and damping, respectively. 1_r is an $r\times r$ identity matrix. u is an n_c-vector of control inputs. To construct $\mathcal{B}_r$ and u, a pair of torquers about the x and y axes is mounted on each of the four corners of the structure $\mathcal{E}$ and on the rigid body $\mathcal{R}$; thus, presently $n_c = 10$. w is taken to be a white noise vector of intensity Q_w due to the actuator disturbances. Since w is due to the torquers, $\mathfrak{D}_r$ is the same as $\mathcal{B}_r$. y is a $2r$-vector of quantities we wish to control. Slope sensors co-located with the torquers are modeled in the n_c-vector z; the rate sensors are not employed. Measurement disturbances are modeled as a white noise n_c-vector v of intensity Q_v. The process of determination of Q_w and Q_v is given in Ref. 1. Dimensions of other matrices can be readily inferred.

Various matrices forming the generic model (13,14) are described below. To lend prominence to the model the corresponding equations are numbered with the prefix M.

$\tilde{\eta}_r$ vector (11×1):

$$\tilde{\eta}_r^T = [\tilde{\eta}_2\ \tilde{\eta}_3\ \tilde{\eta}_4\ \tilde{\eta}_5\ \tilde{\eta}_6\ \tilde{\eta}_7\ \tilde{\eta}_8\ \tilde{\eta}_{12}\ \tilde{\eta}_9\ \tilde{\eta}_{10}\ \tilde{\eta}_{13}] \tag{M1}$$

$\tilde{\omega}_r^2$ matrix (11×11):

$$\mathrm{diag}\{0, 0, 4.1032\mathrm{E}02, 1.0897\mathrm{E}03, 3.2656\mathrm{E}03, 4.9503\mathrm{E}03, 1.0947\mathrm{E}04, 2.6194\mathrm{E}04, 1.4245\mathrm{E}04, 1.7060\mathrm{E}04, 3.1443\mathrm{E}04\} \tag{M2}$$

$\tilde{\mathfrak{B}}_r$ matrix (11×10):

2.1543E+00	0	2.1543E+00	0	2.1543E+00	0
0	8.6173E−01	0	8.6173E−01	0	8.6173E−01
1.0573E−01	2.2062E+00	−1.0573E−01	2.2062E+00	0	0
3.0691E+00	−5.4806E−01	3.0691E+00	5.4806E−01	0	0
4.4194E−01	3.7745E+00	−4.4194E−01	3.7745E+00	0	−2.8571E+00
2.9777E+00	−2.0442E+00	2.9777E+00	2.0442E+00	−3.0626E+00	0
2.1157E+00	4.7510E+00	−2.1157E+00	4.7510E+00	0	0
−8.2330E+00	1.7454E+00	8.2330E+00	1.7454E+00	0	3.1422E+00
2.6296E+00	−4.4189E+00	2.6296E+00	4.4189E+00	0	0
3.4633E+00	3.1066E+00	−3.4633E+00	3.1066E+00	0	6.5744E+00
8.6962E+00	8.3946E−01	−8.6962E+00	8.3946E−01	0	0

2.1543E+00	0	2.1543E+00	0
0	8.6173E−01	0	8.6173E−01
−1.0573E−01	−2.2062E+00	1.0573E−01	−2.2062E+00
−3.0691E+00	5.4806E−01	−3.0691E+00	−5.4806E−01
4.4194E−01	3.7745E+00	−4.4194E−01	3.7745E+00
2.9777E+00	−2.0442E+00	2.9777E+00	2.0442E+00
−2.1157E+00	−4.7510E+00	2.1157E+00	−4.7510E+00
−8.2330E+00	1.7454E+00	8.2330E+00	1.7454E+00
−2.6296E+00	4.4189E+00	−2.6296E+00	−4.4189E+00
3.4633E+00	3.1066E+00	−3.4633E+00	3.1066E+00
−8.6962E+00	−8.3946E−01	8.6962E+00	−8.3946E−01

(M3)

$\tilde{\mathfrak{D}}_r$ matrix (11×10): $\tilde{\mathfrak{D}}_r = \tilde{\mathfrak{B}}_r$ (M4)

$\boldsymbol{P}$ matrix (22×22): $\boldsymbol{P} = \boldsymbol{I}$ (M5)

$\tilde{\mathfrak{M}}_r$ matrix (10×11):

0	−2.1543E−01	−5.5156E−01	1.3701E−01	−9.4363E−01	5.1105E−01
2.1543E−01	0	1.0573E−02	3.0691E−01	4.4194E−02	2.9777E−01
0	−2.1543E−01	−5.5156E−01	−1.3701E−01	−9.4363E−01	−5.1105E−01
2.1543E−01	0	−1.0573E−02	3.0691E−01	−4.4194E−02	2.9777E−01
0	−2.1543E−01	0	0	7.1427E−01	0
2.1543E−01	0	0	0	0	−3.0626E−01
0	−2.1543E−01	5.5156E−01	−1.3701E−01	−9.4363E−01	5.1105E−01
2.1543E−01	0	−1.0573E−02	−3.0691E−01	−4.4194E−02	2.9777E−01
0	−2.1543E−01	5.5156E−01	1.3701E−01	−9.4363E−01	−5.1105E−01
2.1543E−01	0	1.0573E−02	−3.0691E−01	−4.4194E−02	2.9777E−01

−1.1878E+00	−4.3635E−01	1.1047E+00	−7.7664E−01	−2.0987E−01
2.1157E−01	−8.2330E−01	2.6296E−01	3.4633E−01	8.6962E−01
−1.1878E+00	−4.3635E−01	−1.1047E+00	−7.7664E−01	−2.0987E−01
−2.1157E−01	8.2330E−01	2.6296E−01	−3.4633E−01	−8.6962E−01
0	−7.8556E−01	0	−1.6436E+00	0
0	0	0	0	0
1.1878E+00	−4.3635E−01	−1.1047E+00	−7.7664E−01	2.0987E−01
−2.1157E−01	−8.2330E−01	−2.6296E−01	3.4633E−01	−8.6962E−01
1.1878E+00	−4.3635E−01	1.1047E+00	−7.7664E−01	2.0987E−01
2.1157E−01	8.2330E−01	−2.6296E−01	−3.4633E−01	8.6962E−01

(M6)

$\boldsymbol{Q}_w$ matrix (10×10): $\boldsymbol{Q}_w = (2.0\mathrm{E}-14)\boldsymbol{I}$ (M7)

$\boldsymbol{Q}_v$ matrix (10×10):

$$\boldsymbol{Q}_v = \mathrm{diag}\{7.6216\mathrm{E}-07, 1.2195\mathrm{E}-07, 7.6216\mathrm{E}-07, 1.2195\mathrm{E}-07, 7.6216\mathrm{E}-07, 1.2195\mathrm{E}-07, 7.6216\mathrm{E}-07, 1.2195\mathrm{E}-07, 7.6216\mathrm{E}-07, 1.2195\mathrm{E}-07\} \quad \text{(M8)}$$

$\tilde{\boldsymbol{Z}}_c, \tilde{\boldsymbol{\theta}}_x, \tilde{\boldsymbol{\theta}}_y$ vectors: These are truncated and modal-cost-sequence versions of Eq. (9).

$$\tilde{\boldsymbol{Z}}_c^T = [0 \;\; 0 \;\; -2.5218\mathrm{E}-01 \;\; 0 \;\; 0 \;\; 0 \;\; 3.5270\mathrm{E}-01 \;\; 0 \;\; 0 \;\; 0 \;\; -2.6215\mathrm{E}-01]$$

$$\tilde{\boldsymbol{\theta}}_x^T = [2.1543\mathrm{E}-01 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -3.0626\mathrm{E}-01 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$$

$$\tilde{\boldsymbol{\theta}}_y^T = [0 \;\; 2.1543\mathrm{E}-01 \;\; 0 \;\; 0 \;\; -7.1427\mathrm{E}-01 \;\; 0 \;\; 0 \;\; 7.8556\mathrm{E}-01 \;\; 0 \;\; 1.6436\mathrm{E}+00 \;\; 0]$$

(M9)

ζ_r damping matrix (11 × 11):

$$\zeta_r = \text{diag}\,\{0 \;\; 0 \;\; 0.005 \; \ldots \; 0.005\} \qquad \text{(M10)}$$

Scalars in Eq. (6):

$$\tilde{m}_r = 1.6,\ \tilde{I}_{rx} = 0.0021328,\ \tilde{I}_{ry} = 0.013333,\ \beta = 1,$$

$$\tilde{\beta} = \dot{\tau}^2, \dot{\tau} = 5.5885\text{E}-04 \qquad \text{(M11)}$$

$\tilde{\boldsymbol{R}}$ weighting matrix (10 × 10):

$$\tilde{\boldsymbol{R}} = 15.6097\ \boldsymbol{R} \qquad \boldsymbol{R} \text{ is at the user's discretion} \qquad \text{(M12)}$$

Having known all the matrices and the scalars constituting the generic model (13) for the control objective (6) an (sub)optimal controller can be synthesized.

Regarding the quality of the model, we mention that from the standpoint of linear momentum about the z axis and angular momentum about the x and y axes the model is (0.4518, 0.0202, 0.8250) complete, and from the viewpoint of Modal Cost Analysis it is 0.8626 complete; the ideal value of these indexes is unity. The reader is urged to consult Hablani[3] and Skelton et al.[2] to appreciate the significance of the above indexes.

Concluding Remarks

This paper offers a generic model to help evolve a composition of techniques particularly suitable for advanced elastic spacecraft to design a suboptimal controller by using the multivariable control theory. The model deals with both the attitude control of a rigid body and the shape control of an elastic structure.

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Attitude Stabilization of Large Flexible Spacecraft

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I. Introduction

CURRENTLY, there is undeniable interest in very large spacecraft and satellites; the proposed NASA Satellite Solar Power Station is one of many such concepts.[1,2] These spacecraft are mechanically very flexible and, hence, require a large number of vibration modes to describe their behavior. Among the number of theories developed for control of these highly flexible structures is one developed by Balas.[3,4]

In this paper, we apply this theory to a reasonable model of a large spacecraft and develop a controller design to attitude-stabilize the spacecraft, suppress vibrations, and maintain a high level of pointing accuracy. We illustrate the problems of digitally implementing controllers for flexible spacecraft and point out solutions that make the designs implementable.

This paper includes numerical results obtained by Ginter in his M.S. Thesis.[5]

II. Model for Flexible Spacecraft

The generic problem of actively controlling very large nonspinning spacecraft can best be illustrated by considering the structure to be very longitudinally flexible while having much less lateral flexibility (i.e., confining the flexibility to a single plane). This is a reasonable model for many satellites, and it contains the basic elements of all flexible structures. The most flexible such structure is the Euler-Bernoulli beam with free-end conditions. With appropriate boundary conditions, these dynamics are given by the following partial-differential equation:

$$mu_{tt} + EIu_{xxxx} = F \qquad (1)$$

where $u(x,t)$ is the transverse displacement, $F(x,t)$ is the force distribution on the beam, and the beam parameters are the mass density m, the moment of inertia I, the coefficient of elasticity E, and the length L.

The mode shapes $\phi_k(x)$ for this beam model can be obtained as linear combinations of regular and hyperbolic trigonometric functions, and the mode frequencies λ_k can be determined from a transcendental equation.[5] The motion of the beam $u(x,t)$ can be expanded as

$$u(x,t) = \sum_{k=l}^{R} u_k(t)\,\phi_k(x) \qquad (2)$$

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